A Gravitational shock wave generated by a beam of null matter in quadratic qravity

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Abstract. In the present work we approximate an ultrarelativistic jet by a homogeneous beam of null matter with finite width. Then, we study the influence of this beam over the spacetime metric in the framework of higher-derivative gravity. We find an exact shock wave solution of the quadratic gravity field equations and compare it with the solution to Einstein's gravity. We show that the effect of higher-curvature gravity becomes negligible at large distances from the beam axis. We also observe that only the Ricci-squared term contribute to modify the Einstein's gravity prediction. Furthermore, we note that this higher-curvature term contribute to regularize the discontinuities associated to the solution to Einstein's general relativity.

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1. Introduction

The relativistic and ultrarelativistic jets arise as important components in the structure and dynamics involved in several astrophysical scenarios. Among the different types of astrophysical jets, the most energetic ones are potential candidates to give rise to emission of gravitational waves. For example, highly relativistic jets should be associated with some sources of gamma ray bursts (GRBs). In the last few years, observations that indicate the presence of jets in GRBs have been reported by many authors [1]. The expansion of the jet in the burst leads to very high Lorentz factors which is at least of hundreds, but higher values such as $\Gamma \sim 10^3 - 10^6$ are taken into account in modelling the GRB sources [2]. In the present paper, we consider a model in which the gamma Lorentz factor takes the limit $\Gamma \to \infty$. This assumption can be used to solve exactly the gravitational field equations. Then, we can take this analytic solution to represent the gravitational wave generated in a strong burst. To study the impact of an ultrarelativistic jet over the spacetime metric, we start from the extreme situation where the velocity of the particles in the beam is assumed to be equal to the velocity of light. The jet is then represented by a beam of null particles. For jets which start with a small opening angle $\theta_0 \leq 10^{-3} - 10^{-4}$ [1], we assume that the width of the beam remains constant during the first stage of the jet expansion. Then, we calculate the effect of this jet over the spacetime metric in a flat background. We take advantage of the use of exact methods to provide solutions to higher-derivative gravity equations [3] and compute the solution to quadratic gravity field equations.

Quadratic gravity is an example of the higher order theories of gravity which are generally covariant extensions of the general relativity. These theories are constructed by the addition of terms nonlinear in the curvature to the Einstein-Hilbert action. The coupling parameters of the quadratic curvature terms in the Lagrangian must be determined by experiments. Observational constraints on these parameters can be traced out from sub-millimeter tests of the inverse square law [4] or by the bending of light by a gravitational field [5]. Are there a way to obtain an observational constraint on quadratic gravity coupling parameters from gravitational radiation? The application of the traditional methods of ordinary gravitational wave theories, which deals with the linearized version of the gravitational theories, are until the present date, unable to clarify the role played by the quadratic curvature terms in the gravitational radiation [6]. In a recent paper [7] we deal with the linearized version of quadratic gravity treating the higher-curvature terms perturbatively. We conclude that the effects of quadratic curvature terms are so small that cannot be measured by the current interferometric or mass resonant detectors ‡.

In the present paper, we obtain a solution which can be able to explain the effect

‡ The amplitude of a general linearized oscillatory quadratic gravity wave at a given distance from the source differ from the Einstein's linearized wave amplitude by a frequency dependent function. For a wave with frequency of 100 Hz and considering an upper bound of $10^{-4}\mathrm{cm}^2$ for $|\beta|$ [5], this difference becomes $\lesssim 10^{-21}$. This difference cannot be measured by the current gravitational wave detectors.

of higher-curvature invariants on gravitational radiation by dealing with exact solutions to gravitational field equations. We obtain exact shock wave solutions. A gravitational shock wave is truly a discontinuity of the spacetime metric which propagates with the velocity of light in a given direction [8]. The notion of wave amplitude as a finite quantity which varies with the distance from the source cannot be applied here. The gravitational shock wave must be taken instead as a propagating singularity. According to [9] a modification of the gravitational wave detectors would be necessary to observe gravitational shock waves. We do not intend to discuss the detection of gravitational shock waves in the present work. The tidal field of a gravitational shock wave is an issue which we are dealing with and will be presented in a forthcoming paper.

The plane of the present paper is the following. In section 2 we write the field equations for the shock wave metric in the frameworks of quadratic gravity and Einstein's general relativity. This is carried out by assuming that the solution can be described by a pp-wave metric [10]. In section 3 we consider a homogeneous and finite beam of null particles and obtain the gravitational shock wave generated by it. We also compare the solutions obtained in both theories. In section 4 we summarize the principal results obtained and make some comments concerning to the application of these results in astrophysics.

2. The field equations for a plane gravitational shock wave

We start with an impulsive pp-wave metric given by the following line element

$$ds^2 = -dudv + H(x^i, u)du^2 + (dx^i)^2, \tag{1}$$

where

$$H(x^i, u) = f(x^i)\delta(u), \qquad u = t - z, \qquad v = t + z, \tag{2}$$

and x^i (i=1,2) denote the Cartesian coordinates in the plane perpendicular to the wave propagation, which we call transverse coordinates. The metric (1) represents a gravitational shock wave propagating in z direction. This is a variation of the Peres's wave metric [11] and was used by many authors to study wave-like exact solutions to Einstein gravitational equations [3, 12]. The coefficient of δ-distribution is the wave profile function. This function must be determined by the field equations and will depend on the characteristics of the source and on the underlying theoretical model.

The nonvanishing Christoffel symbols for the metric (1) are given by

$$\Gamma^v_{uu} = -f\dot{\delta}, \qquad \Gamma^i_{uu} = -\frac{1}{2}\partial_i f\delta, \qquad \Gamma^v_{ui} = \Gamma^v_{iu} = -\partial_i f\delta, \tag{3}$$

where ∂_i denotes partial derivatives with respect to the coordinate x^i, and the dot denotes the derivative with respect to the u coordinate. The only nonvanishing components of the Riemann tensor, apart from the ones obtained by symmetry properties, are given by

$$R_{iuju} = -\frac{1}{2}\partial_i\partial_j H(x^i, u). \tag{4}$$

The only nonvanishing components of the Ricci tensor for the metric (1) are

$$R_{uu} = -\frac{1}{2}\nabla_{\perp}^2 H(x^i, u), \tag{5}$$

where $\nabla_{\perp}^2$ denotes the Laplacian operator in the transverse space $\{x^i\}$. The curvature scalar R and the elementary quadratic invariants vanish identically

$$R = 0, \quad R_{\alpha\beta}R^{\alpha\beta} = 0 \quad R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = 0. \tag{6}$$

The quadratic gravity theory in four dimensions can be derived from the action

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\{R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + 16\pi G\mathcal{L}_m\}. \tag{7}$$

Since, in four spacetime dimensions, the Gauss-Bonnet invariant can be used to eliminate the $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ term. In the above action, $\mathcal{L}_m$ stands for the presence of matter fields. The units are such that $\hbar = c = 1$. We remark that the gravitational actions built from Lagrangian densities which are arbitrary functions of R are conformally equivalent to general relativity interacting with scalar fields. However, this equivalence cannot be traced for the theory derived from the action (7) due to the presence of the $R_{\mu\nu}R^{\mu\nu}$ invariant [13].

Requiring the action S to be stationary leads to the following field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \alpha \mathrm{H}_{\mu\nu} + \beta \mathrm{I}_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{8}$$

where $\mathrm{H}_{\mu\nu}$ and $\mathrm{I}_{\mu\nu}$ are the operators associated respectively with the R^2 and $R_{\mu\nu}R^{\mu\nu}$ invariants and are given by

$$\mathrm{H}_{\mu\nu} = -2R_{;\mu\nu} + 2g_{\mu\nu}\Box R - \frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu}, \tag{9}$$

and

$$\mathrm{I}_{\mu\nu} = -2R^{\alpha}{}_{\mu;\nu\alpha} + \Box R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\Box R + 2R^{\alpha}{}_{\mu}R_{\alpha\nu} - \frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta}, \tag{10}$$

where $\Box$ refers to the curved space d'Alambert operator. The theory defined by the action (7) leads to the following non-relativistic gravitational potential [14]:

$$V(r) = GM\left\{-\frac{1}{r} + \frac{4}{3}\frac{e^{-m_1 r}}{r} - \frac{1}{3}\frac{e^{-m_0 r}}{r}\right\}, \tag{11}$$

where

$$m_0^2 = \frac{1}{3\alpha + \beta}, \qquad m_1^2 = -\frac{1}{\beta}. \tag{12}$$

This potential reflects the three modes of the linearized theory, one with a vanishing mass which gives the Newtonian force and two massive modes which create Yukawa-type interactions. To obtain an acceptable Newtonian limit we restrict m_0 and m_1 to real values, leading to the, so called, no-tachyon constraints

$$3\alpha + \beta \geq 0, \qquad -\beta \geq 0. \tag{13}$$

Experiments which tests the inverse square law at sub-millimeter distances indicate an upper limit to the absolute value of the α or β parameters of $10^{-4}\mathrm{cm}^2$ [4]. A direct observational constraint on the β parameter can be obtained from the bending of light by the Sun's gravitational field. According to a semiclassical computation of the bending of light in quadratic gravity [5], the higher-curvature coupling parameter β must satisfy the same constraint, namely $|\beta| < 10^{-4}\mathrm{cm}^2$. This bound improves by several orders of magnitude the early accepted value for the upper limit on $|\beta|$ [15].

Now, let us write the field equations for the metric given in (1). By virtue of the identities (6) the operator $\mathrm{H}_{\mu\nu}$ vanishes, ensuring that the square curvature scalar does not contribute to the impulsive wave solution. The field equations (8), reduces to the following linear fourth-order two dimensional partial differential equation:

$$-\frac{1}{2}[\beta\nabla_\perp^4 + \nabla_\perp^2]H(x^i, u) = 8\pi G T_{uu}, \tag{14}$$

where $\nabla_\perp^4 = \nabla_\perp^2\nabla_\perp^2$. In the Einstein gravity, $\alpha = \beta = 0$, the equation (8) reduces to

$$-\frac{1}{2}\nabla_\perp^2 H_{\mathrm{E}}(x^i, u) = 8\pi G T_{uu}(x^i, u). \tag{15}$$

Taking into account the above equation we can rewrite (14) as

$$[\beta\nabla_\perp^2 + 1]H(x^i, u) = H_{\mathrm{E}}(x^i, u) + ah(x^i, u), \tag{16}$$

where $h(x^i, u)$ is a harmonic function of the transverse coordinates x^i and, a an arbitrary constant. This result means that we can add to the final solution a harmonic function of the transverse coordinates. The particular dependence of h on x^i is dictated by the symmetries of the problem and the dependence on the u coordinate is determined by the source term. By making a decomposition of $H(x^i, u)$ as

$$H(x^i, u) = H_{\mathrm{Q}}(x^i, u) + H_{\mathrm{E}}(x^i, u) + ah(x^i, u), \tag{17}$$

where the index Q refers to the purely quadratic part of the solution, we obtain the following second order partial differential equation for $H_{\mathrm{Q}}(x^i, u)$:

$$\left[\nabla_\perp^2 + \frac{1}{\beta}\right] H_Q(x^i, u) = 16\pi G T_{uu}. \tag{18}$$

Thus, the problem of solve the fourth order equation (14) was reduced to the problem of solve the second order equations (15) and (18).

3. The gravitational shock wave associated with a homogeneous beam of null particles

In this section, we compute the solution of equations (15) and (18) when the source is given by a distribution of null particles moving along the same direction. The relevant component of the corresponding energy momentum tensor in polar coordinates is given by [8]:

$$T_{uu} = \lambda\varrho(r, \theta)\delta(u), \tag{19}$$

where $\varrho(r,\theta)$ is the energy density of the distribution of null particles and λ is a dimensionless constant which varies within the range $1 \leq \lambda \leq 2$ and comes from the equation of state

$$P = (\lambda - 1)\varrho, \tag{20}$$

which describes an ultrarelativistic perfect fluid [16].

The line element (1) can be written as

$$ds^2 = -dudv + [f_{\mathrm{E}}(r,\theta) + f_{\mathrm{Q}}(r,\theta)]\delta(u)du^2 + dr^2 + r^2 d\theta^2, \tag{21}$$

where the functions f_{E} and f_{Q} must satisfy the following equations:

$$\nabla^2 f_{\mathrm{E}}(r,\theta) = -16\pi G\lambda\varrho(r,\theta) \tag{22}$$

and

$$\left[\nabla^2 + \frac{1}{\beta}\right] f_{\mathrm{Q}}(r,\theta) = 16\pi G\lambda\varrho(r,\theta). \tag{23}$$

We assume that the energy density is a constant ϱ_0 within a certain radius $0 \leq r \leq R_0$ and zero outside. Thus, the source represents a cylindrical beam of null particles with width R_0. This beam is a simple generalization of a single null particle. We know that the wave profile of a gravitational shock wave generated by a single null particle in quadratic gravity is given by

$$f_0(r) = 8Gp\left[\ln\left(\frac{r}{r_0}\right) + K_0\left(\frac{r}{\sqrt{-\beta}}\right)\right], \tag{24}$$

where p is momentum of the particle, K_0 a modified Bessel function and r_0 some integration constant [12]. This is a continuos function of r which is regular at the origin an diverges logarithmically at $r \to \infty$.

To obtain a solution which generalizes the gravitational shock wave generated by a single null particle we proceed by solving the equations (22) and (23) choosing the integration constants in such a way that the wave profile $f(r) = f_{\mathrm{E}}(r) + f_{\mathrm{Q}}(r)$ fulfill the same boundary and regularity conditions that the single null particle shock wave profile (24). Therefore, $f(r)$ must be *i)* a continuous function of r (even at $r = R_0$, where the source have a discontinuity), *ii)* regular at the origin, namely $f(0) < \infty$, and *iii)* logarithmically divergent at $r \to \infty$.

Let us solve the equation (22). The cylindrical symmetry implies that f_{E} will depend only on the r coordinate. It is easy to see that the solution to (22) can be written as

$$f_{\mathrm{E}}(r) = \begin{cases} -4\pi G\lambda\varrho_0 r^2 + C_1\ln(r) + C_2 & \text{for} \quad r \leq R_0 \\ D_1\ln(r) + D_2 & \text{for} \quad r > R_0. \end{cases} \tag{25}$$

where C_1, C_2, D_1 and D_2 are integration constants.

By intergrating both sides of (22), with $f_{\mathrm{E}}(r)$ given by (25), over a circle of radius $r < R_0$ and applying the divergence theorem to the left-hand side term we get $C_1 = 0$. Now, integrating over a circle of radius $r > R_0$ and applying the divergence theorem to the left-hand side term we obtain $D_1 = -8\pi G\lambda\varrho_0 R_0^2$. The remaining integration

constants C_2 and D_2, are determined by the boundary and regularity conditions. As a regularity condition we impose the continuity of $f_{\rm E}(r)$ at $r = R_0$. This condition imply that

$$D_2 = 16\pi G\lambda\varrho_0 R_0^2 \left(\frac{1}{2}\ln(R_0) - \frac{1}{4}\right) + C_2.$$

The constant C_2, which gives the value of $f_{\rm E}$ at $r = 0$, is the only arbitrary constant that will be determined by the boundary conditions. The regularity at the origin is satisfied by given any finite value to C_2, Then, for simplicity and without loose of generality we choose $C_2 = 0$. Thus, the solution of equation (22) for any $r \geq 0$ is given by

$$f_{\rm E}(r) = \begin{cases} -4\pi G\lambda\varrho_0 r^2 & \text{for} \quad r \leq R_0 \\ -8\pi G\lambda\varrho_0 R_0^2 \left[\ln\left(\frac{r}{R_0}\right) + \frac{1}{2}\right] & \text{for} \quad r > R_0. \end{cases} \tag{26}$$

Let us turn out to the purely quadratic gravity part of the wave profile, $f_{\rm Q}(r)$. Integration of the equation (23) leads to the following result [18]:

$$f_{\rm Q}(r) = \begin{cases} A_1 K_0\left(\frac{r}{b}\right) + A_2 I_0\left(\frac{r}{b}\right) - 16\pi G\lambda\varrho_0 b^2 & \text{for} \quad r \leq R_0 \\ \\ B_1 K_0\left(\frac{r}{b}\right) + B_2 I_0\left(\frac{r}{b}\right) & \text{for} \quad r > R_0, \end{cases} \tag{27}$$

where I_ν and K_ν are modified Bessel functions, $b \equiv \sqrt{-\beta}$ and A_1, A_2, B_1 and B_2 are integration constants.

To obtain a solution $f(r)$ regular at the origin and logarithmically divergent at $r \to \infty$, we set $B_2 = 0$ and $A_1 = 0$ in (27). As we want that $f(r)$ be a continuous function of r, $f_{\rm Q}(r)$ must be continuous at $r = R_0$. This condition is fulfilled by taking

$$A_2 = 16\pi G\lambda\varrho_0 b R_0 K_1\left(\frac{R_0}{b}\right)$$

and

$$B_1 = -16\pi G\lambda\varrho_0 b R_0 I_1\left(\frac{R_0}{b}\right).$$

Thus, $f_{\rm Q}(r)$ takes the following form

$$f_{\rm Q}(r) = \begin{cases} 16\pi G\lambda\varrho_0 \left[R_0 b K_1\left(\frac{R_0}{b}\right) I_0\left(\frac{r}{b}\right) - b^2\right] & \text{for} \quad r \leq R_0 \\ \\ -16\pi G\lambda\varrho_0 R_0 b I_1\left(\frac{R_0}{b}\right) K_0\left(\frac{r}{b}\right) & \text{for} \quad r > R_0, \end{cases} \tag{28}$$

The solution to equation (14) with the source term (19) is given by

$$H(r,u) = [f(r) + a h(r)]\delta(u), \tag{29}$$

where $f(r) = f_{\rm E}(r) + f_{\rm Q}(r)$ is the wave profile and $h(r)$ is a harmonic function. The cylindrical symmetry implies that $h(r) = \ln(r)$. Note that $h(r)$ diverges at $r = 0$ and

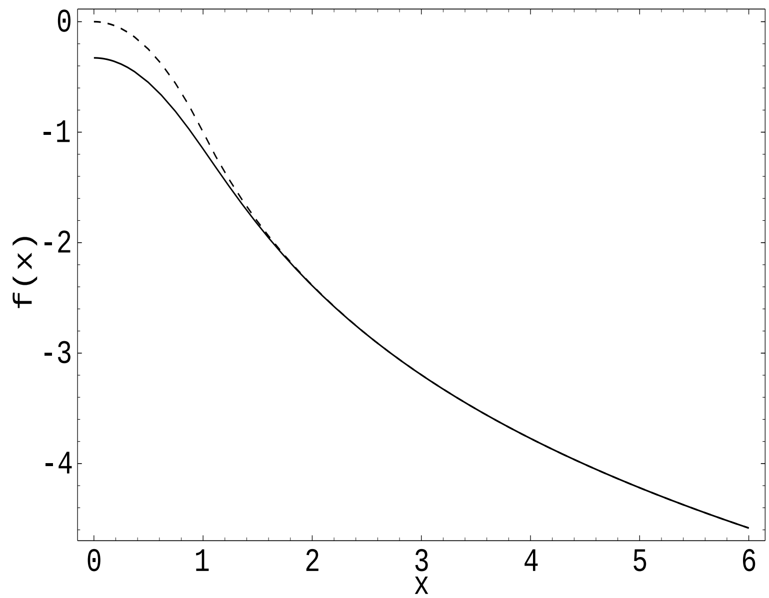

Figure 1. A comparison between the wave profiles in Einstein's gravity (dashed line) and for quadratic gravity (solid line). To clarify the difference of the predictions given by Einstein and quadratic gravity, we set $4\pi G\lambda\varrho_0 = 1$, $R_0 = 1$ and plot the $f(r)$ curve for $\sqrt{-\beta} = 0.3$. We note a non zero residual value at $r = 0$ for the quadratic theory and, we als note that the two curves becomes indistinguishable after a few R_0 distance.

$r = \infty$. If we choose $a = -8\pi G\lambda\varrho_0 R_0^2$ the wave profile approaches to a constant value of $a/2$ as $r \to \infty$. However, the solution will diverge at $r = 0$ unless that we put $A_1 = -a$. But, this choice violate the previous choice, $A_1 = 0$, and implies in the lack of continuity of $f_{\rm Q}(r)$ at $r = R_0$. Therefore, to fulfill the boundary and regularity conditions chosen for $f(r)$ we must set $a = 0$ in (29). The final result diverges logarithmically at $r \to \infty$ and resembles the wave profile for a gravitational shock wave generated by a single null particle [12]

The figure 1, shows the r dependence of the wave profiles $f_{\rm E}(r)$ (dashed line) and $f(r) = f_{\rm E}(r) + f_{\rm Q}(r)$ (solid line).
The non zero value of $f(r)$ at $r = 0$ is given by

$$f(0) = 16\pi G\lambda\varrho_0 \left[bR_0 K_1\left(\frac{R_0}{b}\right) - b^2 \right]. \tag{30}$$

This quantity results from the Ricci-squared term in the gravitational Lagrangian. It carries the information of the beam width R_0 and tends to zero as $R_0 \to 0$. We also observe that the wave profile obtained for quadratic gravity agrees with the prediction of Einstein's gravity at large distances from the beam axis.

A comparison between the first and second derivatives of the wave profiles obtained in the two theories shows that the higher curvature terms of quadratic gravity contributes to smooth out the r dependence of these functions at $r = R_0$. This "smoothing" property of quadratic gravity is related to the fact that the field equations

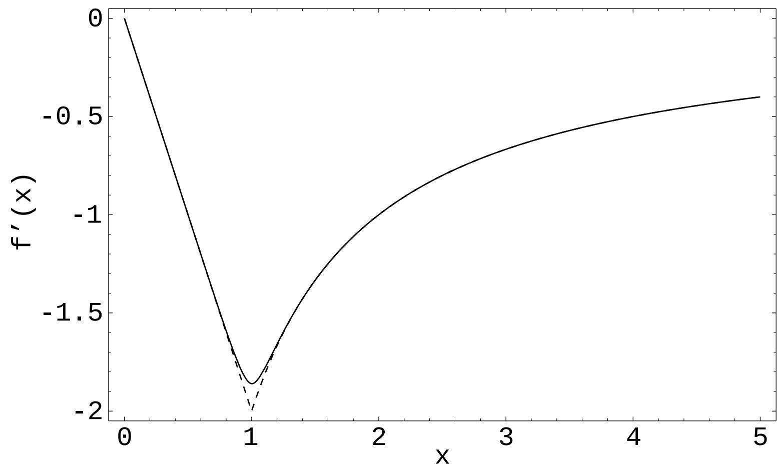

Figure 2. In this figure we plot $f'(r)$ (solid line) and $f'_{\rm E}(r)$ (dashed line) as function of the polar coordinate r. The units are the same used to plot figure 1, but here we set $\sqrt{-\beta} = 0.07$ to compute $f'(r)$. We note the softness of the quadratic gravity result at $r = R_0$ in comparison to the Einstein's gravity one.

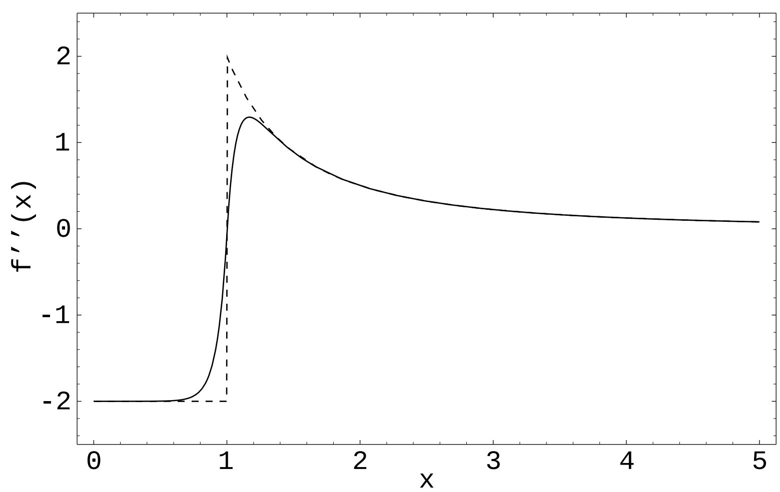

Figure 3. Here are depicted the $f''(r)$ (solid line) and $f''_{\rm E}(r)$ (dashed line) as function of r using the units of figure 1 with $\sqrt{-\beta} = 0.07$. We note that the quadratic gravity curve is smooth at $r = R_0$ while the Einstein's gravity curve shows a discontinuity at this point.

are of fourth order in quadratic gravity, whereas they are of second order in Einstein's Gravity.

In figure 2 we plot the first derivative with respect to r of the wave profiles in both theories. The dashed line represents $f'_{\rm E}(r)$ and the solid line represents $f'(r)$ (the *prime* denotes the derivative with respect to r). Figure 3 shows the r dependence of the second derivatives of the wave profiles. We note the continuity and softness of $f''(r)$ at the point in which $f''_{\rm E}(r)$ has a discontinuity. Therefore, we can conclude that the quadratic curvature component associated to the Ricci-squared term in the gravitational

action smooths out the functional dependence of the derivatives of the wave profile. Furthermore, this higher-curvature term contribute to remove the discontinuity of the second derivative of the Einstein's gravity wave profile at $r = R_0$.

We must bear in mind that a gravitational shock wave cannot be treated in the same way as a linearized gravitational wave. The effect of a gravitational shock wave on test particles is a subject that we will treat in an incoming paper. Nevertheless, we can remark that the test particles would be be sensitive to the analytic proprieties of the wave profile function [17]. Obviously, these properties remains the same whatever the particular values assumed for the β parameter. Thus, even at the limit of $\sqrt{-\beta} \to 0$ the quadratic gravity could in principle be distinguished from Einstein's gravity.

4. Final remarks

Let us now summarize the main results obtained in our investigation and give a suggestion for application of these results in astrophysics.

The first remarkable result is that the R^2 quadratic invariant does not contribute to the wave solution. The only contribution, which comes from quadratic part of the theory, comes from the Ricci-squared term in the quadratic gravity action. The effect of this term over the gravitational wave emission is to regularize the discontinuity associated to the Einstein's gravity solution. We recall that at large distances from the beam axis the predictions derived in both theories coincide.

The gravitational shock wave is very different as compared to an ordinary gravitational wave [9]. The geodesics and the geodesic deviations in spacetimes of gravitational shock waves have been studied in [17]. The authors have showed that the behavior of small geodesic deviations will be dependent on the combinations of the wave profile and theirs derivatives in the transverse space with products and powers of the Dirac δ distribution and the "kink" function, defined by $u\theta(u)$, where $\theta(u)$ is the step function. Therefore, the effect of a gravitational shock wave on test particles will be dependent on these results. We consider that the issue of observation of gravitational shock waves deserves a separated investigation and we intend to do so in another paper, now in preparation, to appear elsewhere.

An important question remains to be discussed. Can the model studied in present the work be addressed to the study of astrophysical sources? We know that some GRBs are generated in the expansion of ultrarelativistic jets [1, 2]. Then a cylindrical beam of null particles can be used as a frist approximation to an ultrarelativistic jet. In this approximation an exact shock wave solution was obtained. We compute the wave forms in the framework of Einstein's and quadratic gravity. If a gravitational shock wave takes place in a GRB scenario it may have important consequences to the physical process at the source. According to [19] a gravitational shock wave would be relevant to the high energy scattering of quantum particles. Therefore it can be espected that the gravitational shock wave contributes to the dynamics of a GRB source and to the composition of the ejected material. Furthermore, once a gravitational

shock wave related to a GRB can be detected, the results derived in the present work could contribute to the understanding of the ultrarelativistic jets features from their gravitational shock wave profiles.

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